# Difference Pattern Beam Steering of Coupled, Nonlinear Oscillator Arrays

T. Heath

***Abstract*—In this letter, it is shown how to extend York's phase-shifterless beam steering technique to difference pattern scanning for monopulse applications. By making a simple modification to the coupling between the central elements of a one-dimensional array, the effective equivalence of the steered sum and difference problems is established.**

***Index Terms*—Beam steering, coupled oscillators, difference patterns, monopulse, synchronization.**

## I. Introduction

QUASI-OPTICAL arrays of coupled, nonlinear oscillators show promise of realizing low-cost, low-loss, compact devices operating at millimeter wavelengths. Exploiting the interactions between array elements has enabled the elimination of lossy components such as phase shifters and corporate feeds while maintaining a coherent, controllable radiation pattern. In 1993, Liao and York proposed and demonstrated a novel, phase-shifterless beam steering method [1], [2] that relied on the synchronization properties of coupled, nonlinear oscillators. Although alternative schemes such as inter-injection locking had been proposed [3], [4], York's technique did not require the use of external signals. Simply by detuning the two end oscillators' natural frequencies relative to that of the interior oscillators, they were, in principle, able to establish phase gradient values between ±90° regardless of the number of array elements. In addition, York and collaborators developed a discrete, nonlinear model to describe the array dynamics [1]–[5]. Recently, the discrete, nonlinear phase model had been shown to be analytically tractable for certain interesting cases of the beam steering problem [6], [7]. However, this analytic and experimental progress has been limited to steering the sum pattern. To date, difference pattern beam steering using small arrays of coupled, nonlinear oscillators has relied on injection locking with external signals [8]. Whereas sum patterns are crucial to target acquisition, difference patterns are important for accurate tracking.

This paper demonstrates the possibility of using York's beam steering method to scan difference patterns for monopulse applications. To this end, a simple modification to the coupling between the two central elements is required. In addition, the stability properties of the sum and difference pattern solutions are shown to be identical.

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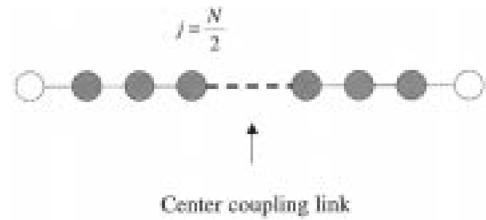


Fig. 1. One-dimensional array with nearest neighbor coupling. The two "fictitious" elements are represented by the dashed, unfilled circles. Connecting the two middle elements [$j = N/2$ and $j = (N/2) + 1$], the central coupling link is prominently displayed.

## II. Sum Pattern Beam Steering

To illustrate the similarity between the sum and difference pattern steering problems, a brief review of sum pattern results is provided first. The system under consideration is a 1-dimensional chain of $N$ nonlinear oscillators with nearest-neighbor coupling. Equations describing the time evolution of the oscillator phases are given by [5]

$$\begin{aligned}\dot{\phi}_j =& \omega_j + k[\sin(\phi_{j+1} - \phi_j + \Phi_{j,\,j+1}) \\ &+ \sin(\phi_{j-1} - \phi_j + \Phi_{j,\,j-1})]\end{aligned} \quad (1)$$

where $j = 1, \ldots, N$ and where the "fictitious" elements $\phi_0$ and $\phi_{N+1}$ have been introduced for notational convenience (Fig. 1). Since the two end elements $\phi_1$ and $\phi_N$ have only a single nearest neighbor, the array boundary conditions are

$$\begin{aligned}\phi_0 &= \phi_1 - \Phi_{1,\,0} \\ \phi_{N+1} &= \phi_N - \Phi_{N,\,N+1}\end{aligned} \quad (2)$$

thereby eliminating the appropriate terms appearing in (1). The coupling strength, coupling phase and oscillator natural frequency are represented by $k$, $\Phi$, and $\omega$, respectively.

For sum pattern beam steering, a spatially uniform phase gradient solution, $\phi_j = \phi_1 + (j-1)\theta$, to (1) is desired. Substituting such a solution into (1)

$$\dot{\phi}_j = \omega_j + k[\sin(\theta + \Phi_{j,\,j+1}) - \sin(\theta - \Phi_{j,\,j-1})] \quad (3)$$

where

$$\Phi_{i,\,j} = \begin{cases}\theta, & (i,\,j) = (1,\,0) \\ -\theta, & (i,\,j) = (N,\,N+1) \\ \Phi, & \text{otherwise.}\end{cases} \quad (4)$$

Demanding (for simplicity) a time-independent phase gradient (i.e., $\dot{\theta} = 0$) and solving for the oscillator natural frequencies, the following conditions must be satisfied in order to establish a spatially uniform phase gradient across the array

$$\omega_j = \omega + k[\delta_{1,j} \sin(\theta + \Phi) - \delta_{N,j} \sin(\theta - \Phi)] \tag{5}$$

where $\omega$ denotes an arbitrary reference frequency and

$$\delta_{i,j} = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases} \tag{6}$$

Equation (5) describes how to beam steer the coupled oscillator array by actively manipulating only two of the $N$ oscillator natural frequencies. As a result, exploiting the dynamical interactions between array elements has led to a significant reduction in the number of controls required for beam steering. The viability of this approach has been demonstrated in several experiments [2], [9]–[13].

In order to be a practical beam steering method, the desired solutions must be robust with respect to small perturbations, i.e., they must be stable solutions of (1). This limits the range of physically realizable scan angles. Substituting perturbed solutions, $\phi_j = \phi_1 + (j-1)\theta + \eta_j$ where $\eta_j \ll 1$, into (1), a set of coupled, linear differential equations describing the evolution of the perturbations is obtained

$$\dot{\eta}_j = a\eta_{j+1} + b\eta_j + c\eta_{j-1} \tag{7}$$

where

$$\begin{aligned} a &\equiv k\cos(\theta + \Phi) \\ c &\equiv k\cos(\theta - \Phi) \\ b &\equiv -(a + c) \end{aligned} \tag{8}$$

and the array boundary conditions imply $\eta_0 = \eta_1$ and $\eta_{N+1} = \eta_N$. The eigenvalues of (7) describe how the perturbations along and away from the desired solution evolve in time. A stable solution requires $N - 1$ eigenvalues with negative real parts. Closed-form expressions [6], [7] for the eigenvalues of (7) are given by

$$\lambda_n = -2k\cos\theta\cos\Phi\left[1 - \cos\left(\frac{\pi n}{N}\right)\sqrt{1 - \tan^2\Phi\tan^2\theta}\right] \tag{9}$$

where $n = 1, \ldots, N-1$. Inspection reveals that (if $k\cos\theta > 0$) the range of stable, spatially uniform phase gradients are $|\theta| < \pi/2$ which, for half-wavelength spacing between the array elements, implies a maximum scan range of $\pm 30°$ off broadside.

## III. Difference Pattern Beam Steering

For a difference pattern, there must be a $\pi$ phase shift between the two halves of the array. This is in addition to whatever uniform phase gradient must be imposed for beam steering (Fig. 2). In what follows, it is assumed that the coupled oscillator array possesses an even number of elements.

Steered difference pattern solutions to (1) are given by

$$\phi_j = \phi_1 + (j-1)\theta + h_j \tag{10}$$

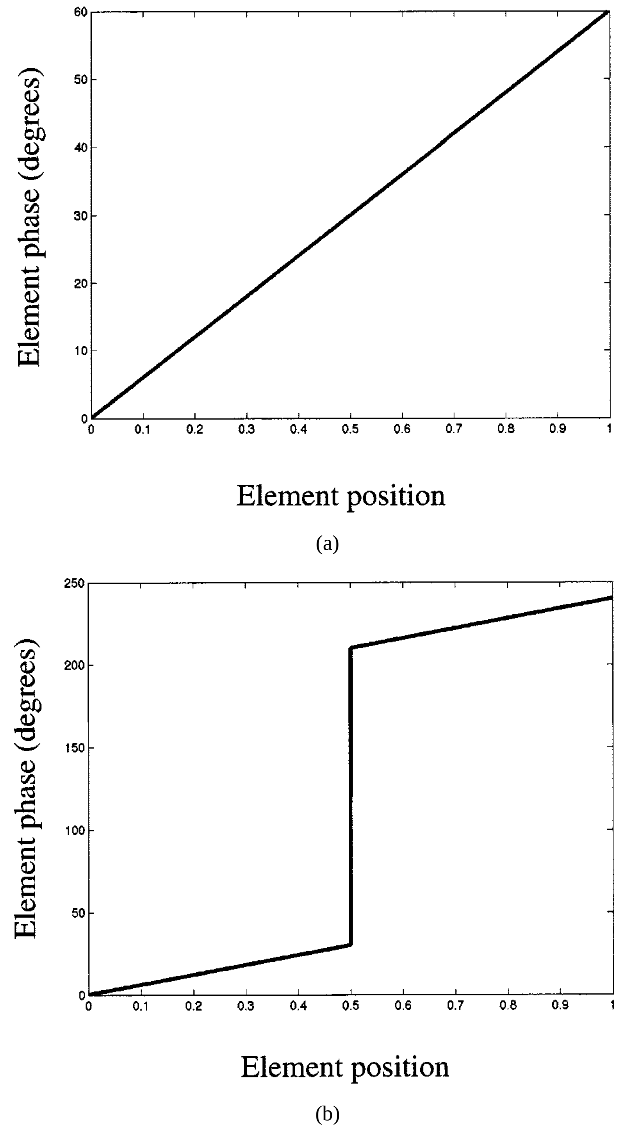


Fig. 2. Phase distribution across the array for a steered (a) sum and (b) difference pattern. Note the difference in vertical scale; the two distributions share the same slope. The array length has been normalized to unity.

where

$$h_j = \begin{cases} \pi, & j > \dfrac{N}{2} \\ 0, & j \leq \dfrac{N}{2}. \end{cases} \tag{11}$$

Substituting these solutions into (1) and using some basic trigonometry

$$\begin{aligned} \dot{\phi}_j = \omega_j + k\,\big[&(-1)^{\delta_{j,N/2}} \sin(\theta + \Phi_{j,j+1}) \\ &- (-1)^{\delta_{j,(N/2)+1}} \sin(\theta - \Phi_{j,j-1})\big] \end{aligned} \tag{12}$$

where the boundary conditions remain

$$\Phi_{i,j} = \begin{cases} \theta, & (i,j) = (1,0) \\ -\theta, & (i,j) = (N, N+1). \end{cases} \tag{13}$$

Apart from a change in sign for certain terms in the $j = N/2$ and $j = (N/2) + 1$ equations, (12) is identical to those arising

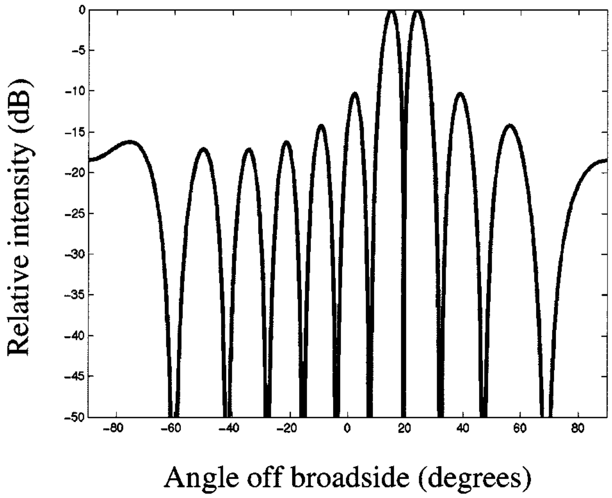


Fig. 3. Steered difference pattern for a $N = 20$ element array with half-wavelength spacing. Additional parameter values were: $\theta = \pi/3, k = 1$, and $\Phi = 0$ (except, of course, for the coupling phase of the central link, which was $\Phi = \pi$). The oscillator natural frequencies were adjusted according to (5). In integrating (1), random initial conditions were used.

in the steered sum pattern analysis, i.e., (3). A simple change to the coupling phase of the center link

$$\Phi_{(N/2)+1,\,N/2} = \Phi_{N/2,\,(N/2)+1} = \Phi + \pi \tag{14}$$

(while $\Phi_{i,\,j} = \Phi$ otherwise) renders (3) and (12) exactly identical. Consequently, the desired difference pattern solution can be beam steered by the same scheme used for beam steering the sum pattern, i.e., by detuning the end elements according to (5). Fig. 3 demonstrates this for a 20-element array.

What remains is to show that the sum and difference pattern solutions share the same stability properties and, therefore, the same scan range. Perturbations to the difference pattern state evolve according to

$$\dot{\eta}_j = a'\eta_{j+1} + b'\eta_j + c'\eta_{j-1} \tag{15}$$

where

$$a' \equiv (-1)^{\delta_{j,\,N/2}} k \cos(\theta + \Phi_{j,\,j+1}) \tag{16}$$

$$c' \equiv (-1)^{\delta_{j,\,(N/2)+1}} k \cos(\theta - \Phi_{j,\,j-1}) \tag{17}$$

$$b' \equiv -(a' + c'). \tag{18}$$

Using the above definitions of the coupling phases for the difference pattern, it is straightforward to show that (15) is identical to (7). Thus, the stability matrix associated with the steered difference pattern solutions is the same as that encountered in the steered sum pattern problem. Consequently, all of the previous results (relaxation rates, scan range, bifurcation points, etc.) of the sum pattern stability analysis holds for the difference pattern as well.

## IV. Conclusion

It was shown that a difference pattern can be steered according to the York method by introducing a $\pi$ phase shift to the central coupling link of a one-dimensional coupled oscillator array having an even number of elements. Moreover, the stability properties of the steered sum and difference pattern solutions were shown to be identical; consequently, they share the same range of stable scan angles and settling rates. This work extends the capabilities of York's phase-shifterless beam steering method to include monopulse applications. Although this paper focused on adjusting the coupling phase, reversing the sign of the central link's coupling strength (i.e., altering $k$ instead of $\Phi$) is an alternative, equivalent approach. Efforts are currently underway to extend these results to two-dimensional arrays incorporate sidelobe reduction via the oscillator amplitude dynamics [14], [15], "hopping" between sum and difference patterns and time-dependent scanning.

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